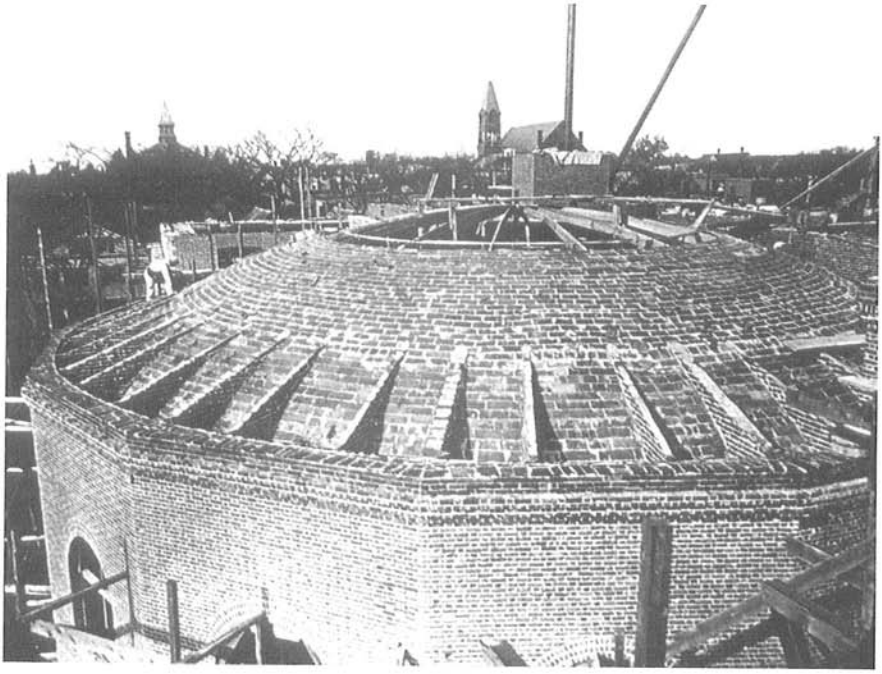

A timbrel dome in construction. The wooden planks at the top are not centering but serve only as a guide to control the geometry during the construction. Note the transverse walls on the springing; they may be used to build the roof, but they are also necessary from a structural point of view. East Boston High School, Boston, Massachusetts, 1899; Architects Brown and Moses; dome design and construction by Rafael Guastavino (Photo courtesy Avery Library, Columbia University)

THE MECHANICS OF TIMBREL VAULTS: A HISTORICAL OUTLINE

Santiago Huerta[1]

Timbrel vaults are masonry vaults, with a good strength in compression, and can be constructed with remarkable thinness and without the use of formwork. Known in the fourteenth century and commonly constructed by the sixteenth, until the middle of the nineteenth century, timbrel vaulting was used vaulting in churches, floor systems and staircases. At the end of the nineteenth century Rafael Guastavino exported the method to the United States, where it was used in many important buildings. This paper examines the development of the theory of timbrel vaults from Espie in the eighteenth century, through Bails and Fornés in the nineteenth, to Guastavino and Guastavino, Jr. in the twentieth, to the use of Finite Element Methods (FEM) today.

Introduction

Timbrel vaults are masonry vaults made with brick and mortar. Their uniqueness derives from their construction: the bricks are placed flatly, forming one or more layers and they are constructed without centering or other support. The bricks are placed in arches or successive rings to complete the vault (Figure 1). During construction, the bricks are supported by the adhesion of the fast-setting mortar to the completed courses, or to the bordering walls. There is no formwork, but guides are used to control the geometry of the vault, particularly for large vaults or for a high-quality finish [Moya 1957; Gulli 2001].The method is analogous to the construction of brick vaults without formwork, which were widely used in Byzantium [Choisy 1883]. These are built with lime mortar, which sets very slowly, and the adhesion of the bricks is achieved by inclining the brick courses, but the construction proceeds in a similar way forming arches or rings. The coincidences suggest a common origin, but the question is still open to further research [González 1999; Mochi 2001; Tarragó 2001].

Timbrel vaults can be constructed with remarkable thinness. Normally two layers of brick tiles are used (about 10 cm in total thickness, including the mortar between the layers), but vaults of only one layer of brick can be found (5 cm thickness). The slenderness ratio, relating the radius of curvature to the span, is typically around 100, but many vaults are even thinner. Also, timbrel vaults with large spans have been built, the greatest being the dome over the crossing in St. John the Divine, New York, with 33 m [Ramazotti 2001].

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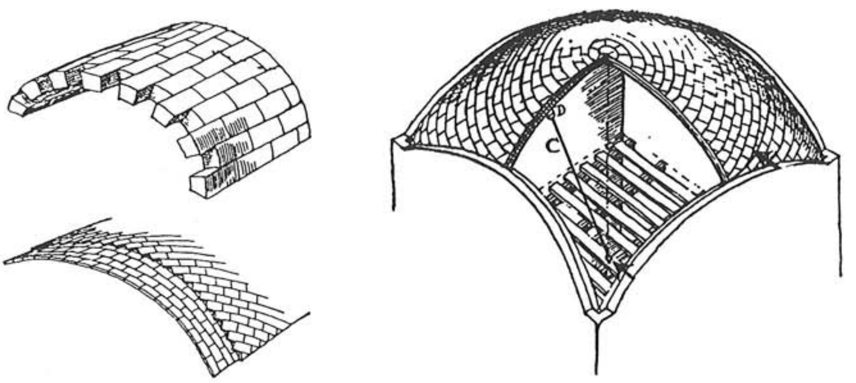

Figure 1. Construction of timbrel vaults. (a, left) Comparison between a timbrel vault and a stone voussoir vault; (b, right) Construction without centering of a timbrel dome. The geometry is controlled by a rod attached to a fixed point [Moya 1957]

Until the middle of the nineteenth century, timbrel vaulting was used for various construction elements: a) to cover the naves of churches. In this case, they could only support their own weight and the occasional loads due to maintenance, and, in general, a timber roof protected these vaults; b) for floor systems; c) for staircases. From the beginning of the nineteenth century, timbrel vaulting began to be used in Spain and France for the construction of roofs and floors of industrial buildings, principally textile factories. The use of Portland cement allowed them to be used as roofs, without the need for an additional roof or other waterproofing methods. In Catalonia towards the end of the nineteenth century and the beginning of the twentieth century the timbrel vault became something of a national symbol [Neumann 1999]. Rafael Guastavino, a Spanish architect, exported the method to the United States at the end of the nineteenth century, and the method developed a greater importance than ever before. 'Guastavino vaulting' was used in many of the most important buildings between 1890 and 1900 in the eastern United States [Collins 1968].

The method of timbrel vault construction is fairly well known and comprehensive bibliographies can be found in the works of Collins [1968], Gulli and Mochi [1995], González [1999] and Huerta et al. [2001]. But the same cannot be said of the structural behaviour of timbrel vaults. The first architectural treatises made no essential distinction between the structural behaviour of timbrel vaults and that of brick or stone vaults. But, at the beginning of the eighteenth century they were viewed by some architects with scepticism, due to a perceived lack of durability and safety. In particular, timbrel vaults were considered to function in a completely different manner than conventional stone or brick vaulting; as we shall see, they were considered to be monolithic and to exert no thrust. Guastavino classified them as "cohesive constructions," as opposed to structures held in place by gravity. Then followed attempts to make elastic analysis, which ended many times in failure. Eventually, in Spain, timbrel vaulting came to be known as "impossible to calculate," and as a result, some of them have been demolished and substituted with more conventional structural systems.

The primary objective of this article is to trace the history of the ideas concerning the structural behaviour of timbrel vaults and, finally, to return timbrel vaults to their place: timbrel vaults are masonry vaults.[2] Like any other masonry structure, they have little resistance to tension, they crack, and they thrust. They are neither monolithic nor cohesive. They can and should be calculated with the same methods used for a vault of masonry. They are also durable if they receive the necessary maintenance.

Traditional timbrel vault design in Spain: Fray Lorenzo de San Nicolás.

The first documents on this type of construction can be traced back to the fourteenth century [Araguas, 1999], and timbrel vaults of the same period still survive in Catalonia. In the sixteenth century they were commonly constructed [Marías, 1991]. They were valued for ease of construction, high strength, and above all, their lighter weight, which allowed for considerable reductions in the size of the supporting walls and buttresses. Explicit reference to the cited advantages can be found in some reports written ca. 1620 during the construction of the Palace of Carlos V in Granada [Rosenthal, 1988].

The most relevant text on the construction and mechanics of timbrel vaulting is the architectural treatise of Fray Lorenzo de San Nicolás, published in Madrid in 1639. Fray Lorenzo, who worked as an architect and built many timbrel vaults, describes the construction of the fundamental types of vaulting (barrel, groined, hemispherical, cloister, etc.) in stone, in brick with radial joints, and in timbrel vaulting. No distinctions are made as to whether one material is better or worse than another. Fray Lorenzo, it seems, considered the three methods to be equally good constructively and he left the selection to the architect in each case. Furthermore, it is revealing that, independent of the material, the vault must be provided with lateral support to carry the thrust to the buttresses. He indicates that it is necessary to fill the haunches for the first third of the vault height and to provide supporting transversal walls, called *lengüetas*, in the second third. (This is the traditional way of construction, which was followed more than two hundred years later by Guastavino; see the introductory figure.)

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[2] The present article is a revised and modified version of Huerta [2001c].

Fray Lorenzo is explicit in the structural role of these elements:

> *...and as you continue constructing, you will layer and solidify the haunches until the first third, and in all of the vaults, placing the* lengüetas, *which rise for another third, and in this manner they will receive the thrust or the weight of the vault* [Fray Lorenzo 1639: fol. 91v].

The fill and transverse walls or *lengüetas* could serve to support the horizontal thrust of soil, but they also had a structural function: they permitted the vault to resist asymmetrical overloads and moving loads.

Fray Lorenzo carried out the calculation of buttress sizes. He gave a series of rules that referred to the standard construction of the period: a church of one nave, sometimes with lateral chapels, covered with a barrel vault (the plan has the form of a Latin cross and over the crossing a dome is built, Figure 2). He proceeded in a systematic form, assigning the dimensions to the vault and considering two possible types of buttress: a continuous wall or a wall with counter-forts. His exposition is so systematic that can be summarized in Table 1.

TYPE OF VAULT	TYPE OF BUTTRESS		
	Wall (uniform section)	Wall with counter-forts	
		Wall thickness	Wall plus counterfort
Stone vault	1/3	1/6	≥1/3
Brick vault: radial joints	1/4	1/7	1/3
Brick: timbrel vault	1/5	1/8	1/4

Table 1. Fray Lorenzo's rules for buttress design. In the treatise the exposition is in the running text, but it is so systematic as to be presented in the form of a table

The ancient builders identified the thrust of the vault with the necessary buttress to resist it. The timbrel vault thrusts less than the brick vault or stone vault, but it thrusts, and requires a system to counter the thrust (Figure 2).

The treatise of Fray Lorenzo gained widespread diffusion in later centuries in Spain (it was still being used by builders at the start of the twentieth century). This is not surprising because the book is exceptional for the number of themes treated and for the clarity of its presentation. Its rules for buttresses are mentioned in many later architectural treatises, for example García Berruguilla

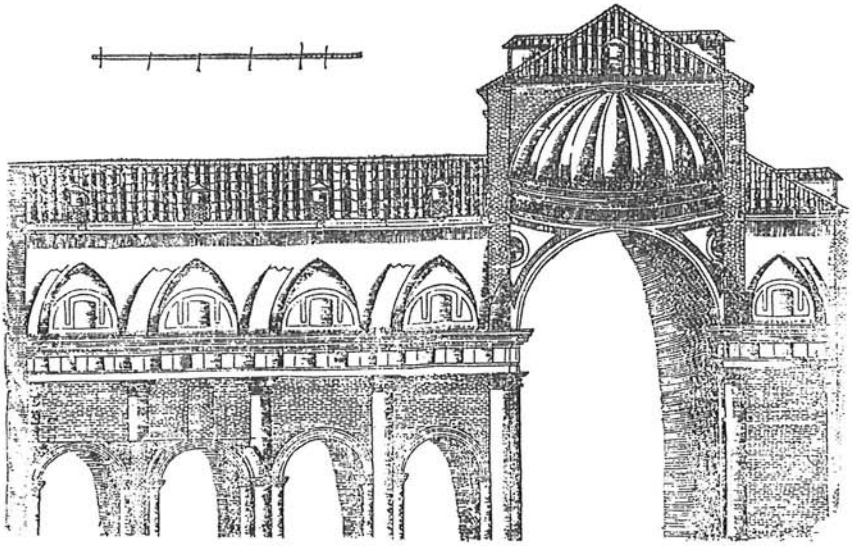

Figure 2. Typical longitudinal section of a Spanish parish church of the seventeenth century. Note the thinness of the dome shell, which may correspond to a projected timbrel dome [Fray Lorenzo 1639]

[1747] and Plo y Camín [1767].

Of course, Fray Lorenzo and the rest of the educated builders of timbrel vault construction knew that once a timbrel vault was finished, the only difference in the structural behaviour compared to the conventional brick or stone vault was the decreased thrust due to the lighter weight. They continued supporting the vaults with buttresses, although fewer were required. Everything else was identical. In particular, the timbrel vaults could be cracked and the pathologies would be identical to those of brick or stone vaults.

The timbrel vault tradition in France: The Comte D'Espie and the myth of 'monolithism'

In France there existed a tradition of timbrel vaulting due to Spanish influence in the region of Rousillon, which Bannister [1968] has studied exhaustively. Around 1700 this tradition passed to the French Languedoc region and in particular, the Duke of Belle Isle built a series of timbrel vaults in his castle, employing the bricklayers of Perpignan. The construction of these lightweight vaults caused a great sensation at the time, and was discussed in the Académie Royale d'Architecture following a paper presented 19 June 1747 by M. Tavenot

[Lemmonier, 1920]. The Académie did not approve of this construction method, which was new to them, but the paper presented by Tavenot is relatively extensive and includes additional information in the appendices.

This type of construction excited the curiosity of a learned nobleman, retired by this point, the Comte d'Espie. He was interested in the possibility of building floors and roof systems in timbrel vaulting, due to its excellent fire resistance. He studied buildings that contained timbrel vaulting and finally constructed a building made with this fireproof construction system. He collected his experiences and opinions in a small book, published in 1754, titled "*Manière de rendre toutes sortes d'édifices incombustibles, ou Traité sur la construction des voûtes, faites avec des briques et du plâtre, dites voûtes plates, et d'un toit de brique, sans charpente, appelé comble briqueté*" (*Manner of constructing all sorts of fireproof buildings, or treatise on the construction of vaults, made with brick and plaster, called flat vaults, and of a roof of brick, without wood, named* comble briqueté.[3]) The book was well received and within a few years was translated and published in English [1756], German [1760], and Spanish [1776]. A second French edition followed in 1776.

Espie begins his book discussing the advantages of timbrel vaulting, stressing its fireproof quality, as well as its lightness and adaptability. He gives also a detailed description of the construction method. But what is of interest for the present study is that he dedicates a chapter to comparing timbrel vaulting with ordinary vaulting: "Parallel des Voutes ordinaires Avec des Voutes Plates" [Espie 1754: 40-58]. He starts by describing the qualitative form in which masonry vaults thrust against the supports. He discusses the influence of the thickness of the vault, its height or rise, and the height of the buttress. He cites Bélidor [1729] in relation to the calculation of thrusts and he warns of the danger of basing projects in practice alone, without some base in theory. Immediately he observes that the cited rules cannot be applied to timbrel vaulting because they are of a different nature and they do not thrust against the walls:

> *Les voutes plates étant d'une nature différente n'ont pas besoin qo'on suivre dans leurs constructions les mêmes regles & les mêmes principes que dans les précédentes; il est donc inutile d'examiner si les murs sont épais ou non. . . . car je ne suis pas de ceux qui croient que ces Voutes poussent les murs* [Espie 1754: 44].

Espie attributes this absence of thrust to the monolithic character of the finished vault, which forms a solid mass due to the good quality of the mortar (plaster) employed. Because it is impossible to form cracks and divide itself the

[3] Lemma [1996] includes a facsimile reproduction and an Italian translation.

vault exerts no thrust:

> *...car le Plâtre lorsqu'il est bien lié avec la Brique fait de toute la Voute entiére un corp massif qui n'a aucun jeu dans ses parties: elles ne se pousseront jamais les unes contre les autres, puisque le tout ensemble ne fait qu'une masse solide qui se contiendra toûjours d'elle-même sans se diviser, pour peu qu'elle soit soutenue* [Espie 1754: 57].

Then he gives a series of observations, made by him personally and others, in support of his monolithic no-thrust theory. In one case he made a load test; in another, he cut away the vault except for the four corners. He made holes in completed vaults to test their resistance. He also writes of a man who built a small vault over a wooden frame, and once the mortar was set, he rolled it around the ranch and hit it with a hammer. They are clearly the reflections of an 'amateur' outside of the tradition of timbrel vaulting. It is interesting to note the 'scientific' approach, trying to obtain conclusions from experiments, but the experiments can be interpreted in several ways and, in fact, many of the tests can be made with normal masonry vaults with the same results. No theory is a direct consequence of a series of experiments.

However, the ideas and experiences gathered by Espie were generally accepted without criticism by later authors. The absence of thrust and the resistance to fire were powerful assertions that created an immediate interest, not only in France but also in the rest of Europe. It was unusual for a work to be translated into Spanish, English and German so quickly. Furthermore, important French and European writers echoed the ideas of this 'new' construction system, basing their comments on Espie's book and relating his ideas. This is the case of Laugier (1755) and of Rieger [1763], but it was particularly important that timbrel vaulting received an extensive treatment in the writing of Blondel and Patte [1771-1777], one of the most influential works of the period. In the sixth volume they dedicated a complete chapter with 40 pages and 7 excellent plates. Undoubtedly, this greatly contributed to the spread of timbrel vaulting (Figure 3).

Two decades later, Rondelet [1802] summarized this information in a section of his *Traité de l'art de bâtir*, including drawings. The treatise of Rondelet was one of the most influential of the nineteenth century. It was translated into German and Italian, and numerous editions were printed. Therefore, at the beginning of the nineteenth century in France there was a theory of timbrel vault construction that was based primarily on the opinions of a learned nobleman who aimed "to serve the community."

In summary, some of the most outrageous ideas about timbrel vaulting (its monolithic nature, absence of thrust, etc.), which spread rapidly throughout Europe, had their origin in the treatise of the Comte d'Espie. These ideas formed the official 'frame of reference' for approaching these structures, and came back to Spain, then under a heavy French influence.

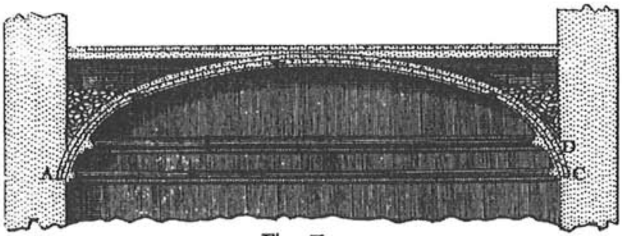

Fig. X.

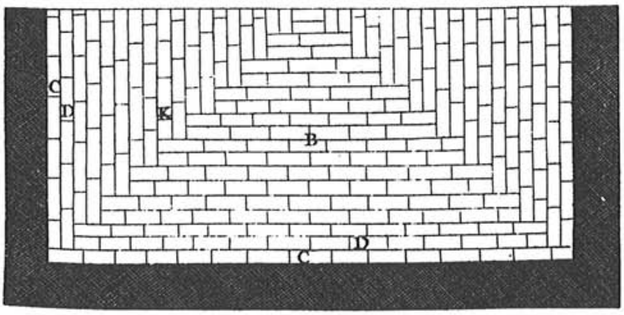

Figure 3. Construction a timbrel vault for the floor of a building. Note the filling on the haunches and the existence of transverse walls to support the floor. In plan the form of bonding of the bricks forming successive arches parallel to the walls [Blondel 1777: vol. 6, Plate 95]

The Spanish edition of Sotomayor and the 'Censure' of Ventura Rodríguez

Joaquín de Sotomayor [1776] translated the Espie's book into Spanish, adding his own opinions and experiences in brackets. More interestingly, the book was preceded by a 'Censure' from Ventura Rodríguez, the official architect of Madrid and one of the most important Spanish architects of the eighteenth century. The beginning captures the tone of the Censure: "We would obtain considerable advantages in the art of building if all of the feasible ideas we propose were as successful in practice as they seem in fantasy."

There follows a sharp critique of the fundamental ideas of Espie: the monolithic nature and the resulting lack of thrusts. Ventura Rodríguez cites various cases of cracking and displacements in completed buildings which demonstrate the thrust of the vaults:

> *Though this supposition [the absence of thrust], or belief, is flattering, it is in spite of the cited experience and cannot be verified, as affirmed by the evident examples that we have in almost all of the Temples of Madrid, whose vaults are timbrel vaults of brick and plaster, of high curvature, and with thick walls, protected by buttresses, whose firmness is a great advantage . . . and we have seen them broken in many places, and with the walls displaced, due to the thrust.*

He insists numerous times in the necessity of providing sufficient support for timbrel vaults and he emphasizes the importance of "firmness", in addition to the 'beauty' and the 'convenience', for if this fails, "all is lost". Clearly, Ventura Rodríguez does not agree with the opinions of Espie and considers them to be dangerous. In fact, Sotomayor, like Espie, was an 'amateur' of the construction method, but not a builder. Ventura Rodríguez, an architect of great experience, saw immediately the mistakes and perils of Espie's 'monolithic', no-thrust theory.

The first scientific experiments in France

Apparently the interest in timbrel vault construction continued in France during the nineteenth century. Historical studies on this theme are lacking and the only evidence we have found is the realization of experiments trying to ascertain the thrust and strength of timbrel vaults: D'Olivier [1837] and Fontaine [1865]. It is remarkable that in both cases the thrust of the vaults is considered (negating implicitly Espie's theory) and that the calculations were made following conventional masonry vault theory. Of particular interest are the large-scale tests to failure described by Fontaine. One of the tests described is on three timbrel vaults with a span of 4 m (and a rise of 0.4 m), spanning between wrought iron I-beams (of 47 cm depth) with a span of 6.25 m, covering a total area of 72 sq m. The test was carried out until failure occurred under a load of 1,250 kg/m^2. In another test on a timbrel vault spanning 3.75 m (again with rise:span ratio of 1:10), the vault carried a load of 2,700 kg/m^2 without failing. Tests of such magnitude were not made in an isolated manner and, most probably, were carried out in the hope of producing fireproof vaults for factories. (In fact the size of the test of the three vaults coincides with the usual plan module for textile factories.)

Spanish treatises of the first half of the nineteenth century: Bails and Fornés

In nineteenth-century Spain, Espie's influence is evident in two later Spanish treatises which treat timbrel vaulting, those of Benito Bails [1796] and Manuel Fornés [1841, 1846]. Bails basically compiled and plagiarized previous French manuals, particularly Blondel/Patte. He dedicates a chapter to timbrel vaulting. Initially he transcribes the corresponding paragraphs of Fray Lorenzo de San Nicolás, but then he copies, translating into Spanish, directly from Blondel/Patte. He is apparently unaware of the contradiction between both texts.

The writing of Fornés is original. First published in 1841 and revised in 1857, Fornés presents the method of building timbrel vaults and makes a new contribution. He sets out in great detail the way of building the principal types of timbrel vaulting: barrel vaulting, staircases, domes, squinch arches, etc. In regard to the thrust, Fornés considers that timbrel vaults thrust, though less, due to their smaller thickness, following the traditional ideas of Fray Lorenzo and Ventura Rodríguez.

However, Fornés knows the ideas of Espie, probably through Bails, and contradictions begin to appear. Thus, in the first part of his treatise he affirms he discusses the geometry and thickness of the vaults from which originates the thrusts and the size of the walls to resist it. But further on he writes: "[the timbrel vault]...covering the work and walls, the material becomes a solid body, equal to the lid of a pot, with no more thrust than its weight" [Fornés 1841: 47]. As with Bails, Fornés seems unaware of his contradictions. However all the projects of vaulted buildings included in his second treatise possess the usual buttressing of masonry buildings.

Rafael Guastavino's theory of 'cohesive construction'

Rafael Guastavino was the first to attempt to formulate a theory that explained, in a scientific form, the structural behaviour of timbrel vaults. To put his work in context it is necessary first to give a brief biographical sketch of his fascinating life. Born in Valencia in 1842, he went to Barcelona in 1861 where he began his studies of building and architecture. By 1866 he had already built apartment house and in 1868 began the construction of the huge Batlló Factory. There he used the timbrel vault technique extensively (Figure 4), and by then he was convinced that the future progress and perfection of masonry construction would be with this type of building. This idea became the objective of his life, his *Leitmotiv.*

Afterwards he built a number of buildings in Barcelona [Bassegoda 2001]. Then, he won an Award in the Philadelphia Exhibition of 1876 and eventually

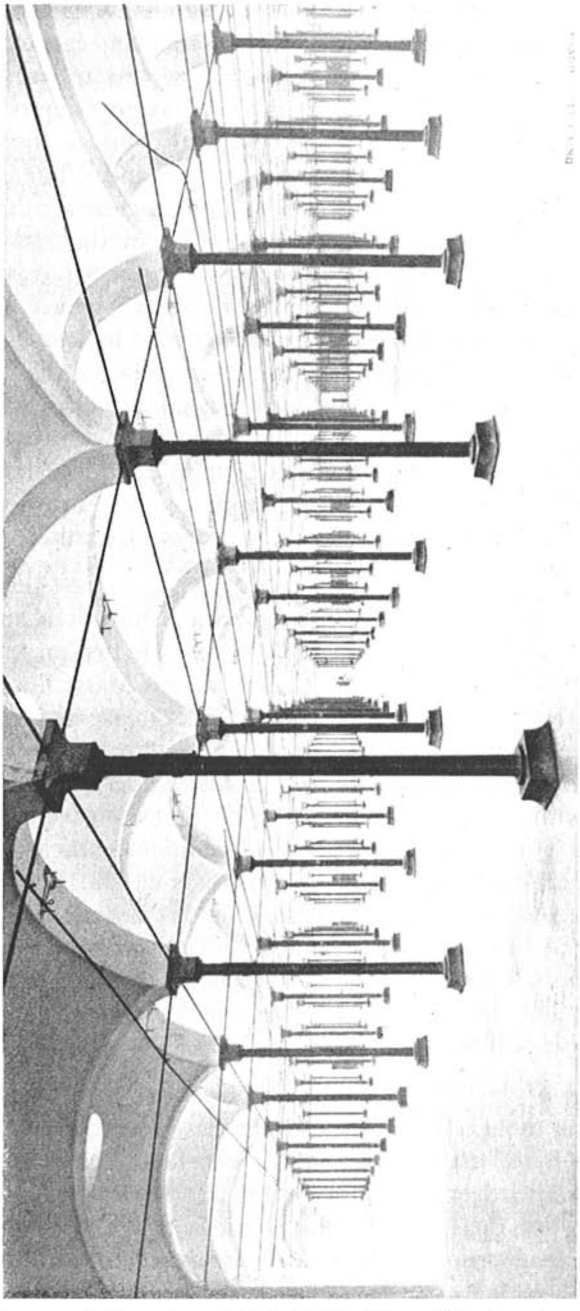

Figure 4. Batlló Factory in Barcelona (1868-1870). View of the interior with an extensive use of timbrel domes (Archivo Histórico de la Diputación de Barcelona)

decided to immigrate to America, arriving at New York in 1881 with his elder son Rafael Guastavino, Jr. After a brief stint as an architect, he decided that the best way to promote the use of timbrel vaulting was to work as a building contractor and in 1889 he established the Guastavino Fireproof Construction Company. The same year he began his first great contract: the building of the vaults of the Boston Public Library (McKim, Mead and White, architects). The lightness and audacity of this new structure aroused great admiration. It should be kept in mind that, before Guastavino, many of the vaults in American buildings were 'false', hanging from a wooden or iron structure; this was cheaper than the usual stone or brick vaults. Timbrel vaults, lighter and constructed without the need of heavy centering, were attractive to many architects. After the Boston Public Library, Guastavino worked for some of the most important architects of this time [Collins 1968]. But it was not at all easy; this type of vault was completely unknown in America and was looked with suspicion for many builders. The first task of Guastavino was, then, to convince American architects and engineers of the strength and high quality of these structures, and also to demonstrate their kinship with great masterworks of architecture, such as the Pantheon in Rome or Hagia Sophia.

Guastavino needed a theory both technical and historical, and he had been conscious of this since his first works in Barcelona. He first presented his ideas at a series of seminars for the Society of Arts at the Massachusetts Institute of Technology in 1889. He also published a series of magazine articles in 1889, and, finally, presented the ideas in a book, *Essay on the Theory and History of Cohesive Construction, applied especially to the timbrel arch,* published in 1892 and reprinted with minor corrections in 1893. (For the genesis of this book see Parks [2001]). Guastavino later published additional articles and conference papers (for a complete bibliography see [Huerta et al. 2001]), as well as another book titled *Prolegomenos on the use of masonry in modern constructions* [1896-1904]. While this last book is fundamental to understanding the architectonic thinking of Guastavino, it does not include new information on his ideas and calculations pertaining to the structural behaviour of timbrel vaulting. Therefore in what follows we will refer to the *Essay* of 1893.

The first part of the *Essay* is autobiographical and in it Guastavino explains the sources of his thoughts. He mentions the classes he received in the School of Architecture in Barcelona from his instructors, Juan Torras and Elías Rogent. Guastavino acknowledges them for calling his attention to this method of construction, which, he later affirms at various places in his book, had been forgotten for a long period of time. This last assertion is extremely doubtful.[4]

[4] This assertion is more than debatable. Timbrel vaulting had had a constant presence in the Spanish construction treatises since the seventeenth century. It is also a significant fact that an

We do not know the content of these classes in Barcelona, but it is likely that the professors presented Espie's ideas of continuity and monolithic nature, which were known in the Spanish treatises of the time [Sotomayor 1776; Bails 1796; Fornés 1841].

Guastavino divided masonry structures into two groups according to their structural behaviour:

> *We will divide construction in general into two classes:*
>
> *First, "Mechanical Construction," or construction by gravity.*
>
> *Second, "Cohesive Construction," or construction by assimilation.*
>
> *The first, is founded in the resistance of any solid to the action of gravity when opposed by another solid. From these conjunctive forces, more or less opposed to one another, results the equilibrium of the total mass, without taking into consideration the cohesive power of the material set between the solids.*
>
> *The second has for a basis the properties of cohesion and assimilation of several materials; which, by a transformation more or less rapid, resemble Nature's work in making conglomerates* [Guastavino 1893: 45].

Timbrel vault construction is cohesive, but it is not the only type of cohesive construction. In the second half of his *Essay* he makes a confusing historical revision. Roman concrete construction is 'evidently' cohesive, but Guastavino also considered Byzantine and Islamic brick constructions as cohesive; besides, he claims the Middle Ages as the era in which the cohesive system truly developed. Also the great domes of the Renaissance are cohesive. In fact, the list of cited buildings includes some of the most notable buildings from different periods and styles: the baths of Caracalla, the Hagia Sofia in Istanbul, the Cathedral of Zamora, the great Cathedral and Baptistery of Florence, St. Peter's in Rome, Sainte-Geneviève in Paris, St. Paul's in London, and two timbrel vaulted domes in Valencia [Guastavino 1893: 26-29]. Apparently, any building constructed in a material with good adhesion of the mortar, including Roman concrete, brick, timbrel vaulting, etc., falls in the category of cohesive construction. No doubt

important part of the treatise by Fornés, published in Valencia in 1841 (2nd ed. 1857), is dedicated to timbrel vaulting. Fornés systematically uses timbrel vaulting in the construction specifications of his "Album de proyectos" of 1846. It seems then, that timbrel vault construction was well known in Valencia in the middle of the nineteenth century, not to mention the extraordinary timbrel domes which existed in the city. As for the rest of Spain, the treatise of Ger y Lóbez, published in Badajoz in 1869, also discussed timbrel vaulting, to the same extent that it discussed vaulting in brick or stone.

Guastavino is looking for historical arguments in favour of timbrel vault construction.

Another point of great importance is his remark about the 'natural' character of cohesive timbrel vault construction. Guastavino was fascinated by the possibility of constructing a solid from many small pieces, in the same way that nature forms conglomerates. He describes his fascination with a visit to the great cave of the Monasterio de Piedra in Spain:

> *The thought entered my mind, while in this immense room . . . that all this colossal space was covered by a single piece, forming a solid mass of walls, foundation and roof, and was constructed with no centres or scaffolding . . .*
>
> *This grotto is really a colossal specimen of cohesive construction. Why had we not built on this system?* [Guastavino 1893: 13; (emphasis mine)]

This passage is the key to understanding the structural thinking of Guastavino. The idea that cohesive construction (including timbrel vaulting) is a 'natural' construction, and furthermore, was 'more rational, durable, and economical', came to him as a revelation and was a driving force for his work throughout his life. As we will see, the cohesive character does not influence the essential behaviour of timbrel vault structures, but the research to improve the cohesion led to an unprecedented perfection of timbrel vault construction.[5] On the other hand, the observation about the monolithic character of structure ("covered by a single piece") echoes the ideas of Espie, which he may have heard in his classes in Barcelona. However, the essential character of timbrel vault construction, the possibility of dispensing with centering, though mentioned, became a matter of secondary importance in the *Essay*.

Advantages of the 'cohesive' timbrel vault

Guastavino explained the differences between gravity construction and cohesive construction in relation to timbrel vaults, aiming to prove the advantages of the latter [Guastavino 1893: 49-57]. He compares a timbrel arch of one layer with a timbrel arch composed of two layers (Figure 5). In the single-

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[5] This is not unusual in the history of the development of science and technology. From time to time, a scientist or artist is led by a false idea, but their enthusiasm helps them to develop other correct ideas which can lead to an advance in the discipline. Koestler (1964) cites, among other examples, the case of Kepler, who had a lifelong obsession with the geometrical harmony of the movement of spheres, discovering laws which shattered the Greek geometrical ideas on the movement of stars.

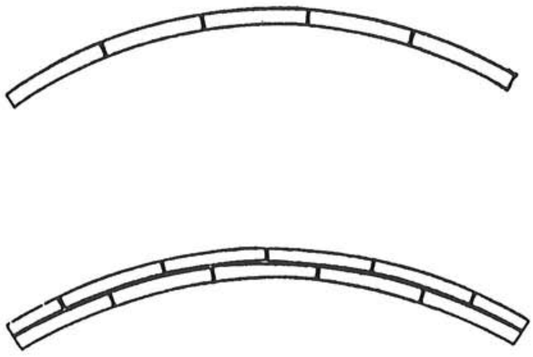

Figure 5. Comparison between a 'mechanical' arch (above) and a 'cohesive arch' (below) [Guastavino 1893]

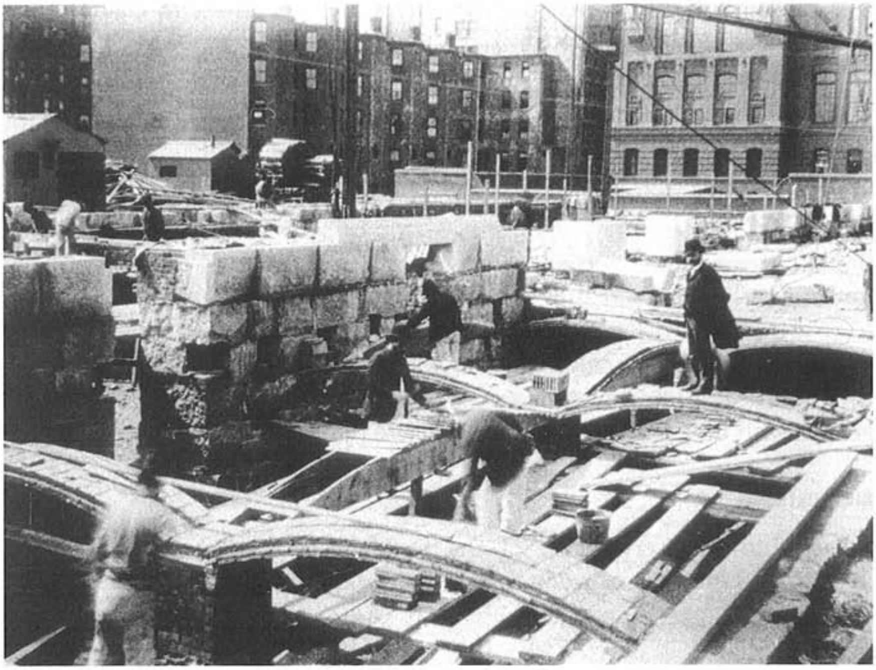

Figure 6. Construction of timbrel arches. The man standing is Rafael Guastavino, demonstrating the strength of these thin arches. Boston Public Library, 1889-1890, McKim, Mead and White, architects (Avery Library, Columbia University)

layer arch there are joints between the bricks, which, he says, function like the voussoir in a traditional gravity arch. The double-layered arch, with mortar between the two layers and with overlapping joints, forms an arch which functions as a cohesive structure, capable of resisting bending moments. The evidence for this assertion is that it is possible to construction barrel vaults spanning twenty ft (6 m) with a thickness of only 3 in (7.5 cm). After only a few hours, the workers can walk on top of the vault safely – and this is indeed a proof of a certain bending resistance. Finally, the form used in construction can be placed as proof that the vault has not deformed. Guastavino attributed great importance to this characteristic of timbrel arches, and it is no coincidence that he photographed himself standing on one of the recently completed timbrel arches of the Boston Public Library (Figure 6).

Guastavino attributed many of the structural advantages of timbrel vaults and arches to the reduction in the number of joints. If it were possible to construct without joints it would be ideal: "It is evident that if we were able to build an arch without joints, it would be the best, as it would have no settlement" [Guastavino 1893: 52]. Once again, Guastavino cites the myth of a monolithic nature. Of course masonry arches and vaults crack due to changes in geometry; this is the only way in which the masonry structures adjust to changes in the boundary conditions.[6]

Guastavino summarizes the advantages of the timbrel arches and vaults in relation to mechanical arches:

- The vertical joints are protected from cracking by the overlapping of joints;
- There are fewer vertical joints;
- There is capacity to resist bending moments.

Of course, massive concrete arches (without reinforcing) are cohesive arches with no vertical joints, but Guastavino discards them due to excessive cost and problems with irregular setting of the concrete [Guastavino 1893: 56].

Load tests

Guastavino was very aware of the problem of convincing the American architects of the merits of timbrel vault construction. Even in Spain, where the

---

[6] The phenomenon is well known since antiquity. The first interpretations of the cracking as a result of support movements appeared in the middle of the nineteenth century. Jacques Heyman made the first systematic studies; see, for example, Heyman [1997].

method had been used for centuries, these structures were often viewed with a lack of confidence. Guastavino's theoretical and historical speculations were necessary, but above all, he had to make scientific tests. Although earlier tests were made in France (see above), Guastavino was unaware of them and made his own tests. The first systematic tests were made in 1887 on timbrel specimens (Figure 7). Later, in 1901, he carried out also structural load tests, and fire tests to demonstrate the strength and "fireproof" nature of timbrel vaulting.

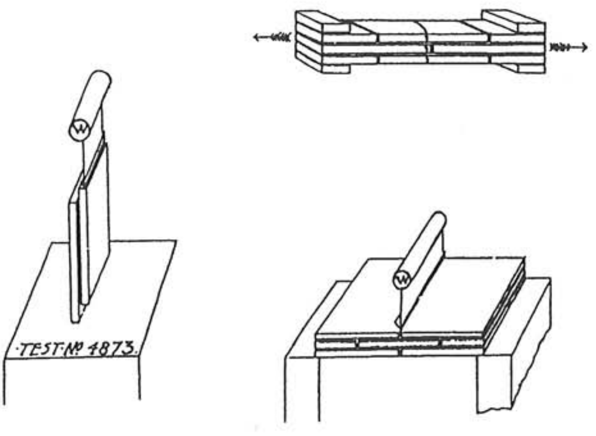

Figure 7. Specimens for the strength tests made by Guastavino: (a, upper right) tension; (b, lower right) bending ; (c, left) shear. [Guastavino 1893]

In the material tests he tried to obtain breaking stress values for compression, tension, shear, and bending. These values could then be used to verify the safety of his vaults by comparing the working stresses with the material failure stress. This of course, was the focus on stress and strength which began with Navier [Heyman 2001]. The results of the tests are summarized in Table 2. It is interesting that he does not mention any attempt to determine the elastic constants, such as Young's modulus or Poisson's ratio.

STRENGTH	N/mm^2
Compression	14.60
Tension	2.00
shear	0.90

Table 2. Mean strength of timbrel specimens [Guastavino 1893]

Figure 8. Load test made by Guastavino on a barrel vault, 1901 (Avery Library, Columbia University)

Figure 9. Load tests made by Guastavino in a staircase under construction, 1903 (Avery Library, Columbia University)

To compare these results with the real structures, he made failure tests of flat timbrel vaults with a rise of 1/10 of the span. One of these tests is shown in Figure 8. The photo is spectacular, and says more in favour of the strength of timbrel vaults than any theory or laboratory test results. In this case, it is clear that these tests played a role as propaganda, which was common at the end of the nineteenth century.[7] On other occasions, load tests were made during construction (Figure 9).

The thrust of timbrel vaults and domes

Practically the only information on the calculation methods used by Guastavino is found in his *Essay*. (The graphic methods of structural analysis can be attributed to Guastavino, Jr., and will be discussed later.) Guastavino treated two typical themes: the flat barrel vault and the hemispherical dome, also flat. To obtain the thrust of a flat arch or barrel vault he gives the following formula (I have modernized the notation) [Guastavino 1893: 59]:

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[7] It is interesting to compare the tests and the pictures with those made by Hennebique on reinforced concrete. See Delhumeau [1999].

$$A(S_{br}) = \frac{Wl}{8f} \qquad [1]$$

where A = cross-sectional area of the vault at the crown per unit length; S_{br} = breaking stress in compression; W = total load (self-weight plus fill and live load) acting on the vault per unit of length; l = span of the vault; f = rise of the vault.

The formula relates the load W with the area A (depth) and the breaking stress of the material for an arch of given geometry. Guastavino treats the equation for the thrust of a parabolic arch under a uniformly distributed load as a given, though his 'demonstration' is difficult to understand. The formula is approximate since the loading is not exactly uniform, but for flat vaults it is sufficiently accurate. Guastavino gives an example application: to calculate the thickness of a vault spanning 15 ft (4.575m), with a rise/span ratio of 1/10, under a uniform load of 250 lbs/ft^2 (12 kN/m^2). The material has a breaking strength of 2120 lbs/in^2 (14.6 N/mm^2). Guastavino considered the permissible working stress to be 1/10 the breaking stress, so the allowable working stress would be 212 psi (1.46 N/mm^2). Entering these values in the formula gives a required thickness of 1.85 in (4.7 cm), which requires two layers of one-inch thick bricks. Of course, a safety factor of ten is excessive even for an irregular material like the masonry of timbrel vaulting. Further on, Guastavino admits that working stresses can be considered at 1/4 or 1/5 the breaking stress [Guastavino 1893: 64].[8] In fact, in masonry structures and in timbrel vault structures, the criterion governing safety is not the strength of the material but the stability of the system. Safety is obtained by giving a sufficient thickness. Perhaps the oscillation between four and ten for the coefficient of safety allowed Guastavino to choose the thickness that seemed adequate to him in each case.

The formula gives the thickness at the crown. The force is larger at the supports and to find the new thickness would require the application of "Dejardin's formula" [Dejardin 1860], stating that the force increases from the

[8] The factor of safety of 10 applied to masonry has an origin. The strength of a masonry element depends on the strength of the stones, the form and dimensions of the mortar joints, and the strength of the mortar, as shown by the tests of Tourtay [1885]. Thus, to derive factors of safety against breaking, one must consider blocks of a certain size. However, the first tests were made with small pieces of stone. Knowing that the strength of the actual element would be much less, the engineers of the nineteenth century took, in an empirical form, the admissible strength of the fabric as 1/10 the breaking strength of the stone. Of course, the rule does not apply to the tests made on small samples, as in the case of Guastavino. This fact led to considerable confusion in the engineering manuals at the end of the nineteenth century, and many times, to absurdly low values of admissible stress in masonry structures.

crown down to the support.[9] Guastavino's calculation is evidently an equilibrium calculation. It obtains a value of thrust, and later checks this value against the strength of the material (normally unnecessary), and also to calculate the value of lateral thrust at the supports. The thrust was usually resisted with masonry buttresses, or more frequently, using a system of wrought iron ties.

However, at the end of the nineteenth century the elastic theory was considered as the best option to analyse masonry arches, and the equilibrium method, although used in practice, was viewed with suspicion by engineers. In all likelihood, Guastavino did not have sufficient training to make an elastic calculation, which required the solution of complicated integrals for even the simplest cases. For this reason he hired a professor of applied mechanics from MIT, Gaetano Lanza [1891], to calculate a table of the elastic stresses in timbrel arches, taking into account the normal force and the bending moment. The table is included at the end of the book without any explanation. This was another attempt to give scientific respectability to the calculation of timbrel vaulting. Comparing the results of his formula with the elastic table, there are not significant differences, but this is logical for flat arches.

Guastavino moves on to domes, which he considered to be an excellent form: "The dome is the genuine form of cohesive construction for ceilings, floors, and roofs, as well as for timbrel arches" [Guastavino 1893: 66]. To calculate the thrust, Guastavino makes another approximation, reasoning (incorrectly) in a geometrical manner to compare the areas of a sphere and a half cylinder of the same radius developed in plan. In effect cutting the cylinder as indicated in the figure and joining the wedges forms a polygonal dome approximating a sphere. Seeing the plan, Guastavino considered that the weight of the dome is one half the weight of the corresponding barrel vault, and therefore, the thrust would be half. In fact, the weight is different and also changes the position of the centres of gravity, but to consider the thrust of a dome as half the thrust of a vault is a safe assumption, since the thrust is normally smaller (for a hemispherical dome the thrust is close to one third). The idea comes from Frézier [1760: 3, 406], later spread through some architectural manuals, and Guastavino could have learned

[9] Dejardin's book was very popular in the second half of the nineteenth century, and it included rules and observations of practical interest. The rule to obtain the thickness variation of an arch is very old. It originates in the equilibrium analysis by La Hire [1695] of a semicircular arched formed by rigid voussoirs without sliding. To maintain equilibrium, the condition is that the weight of the voussoirs (i.e., thickness) must vary with the inverse of the cosine. Frézier was the first to propose arches of variable section on the basis of this. Later it became common in masonry arch bridges, but not in arches for building construction [Huerta, 1990]. In the case of a timbrel vault, which supports moderate loads, it does not seem necessary to give such low working stresses. However, the choice is logical if one considers that it is a problem of material strength (though this is not the case) as Guastavino did.

this in his classes in Barcelona. For flat arches, Guastavino's formula [1] gave a good approximation of the thrust, but in this case there could be significant variations, as Guastavino himself recognized: "We do not pretend to give an absolute mathematical formula, but a practical one, which is sufficient to guarantee the building safety" [Guastavino 1893: 68, 72].

Guastavino is using the usual, simple formulae for the thrust of masonry, voussoir, arches, vaults and domes, but he is apparently unaware of this. On the other hand, after calculating the thrust in the manner indicated, without considering bending moments, he contradicts himself: "We consider our arch not as an arch with voussoirs, but as a single cast arch, working as a solid piece of arched stone or iron" [Guastavino 1893: 69]. He later states, "We are here considering the dome as not one of voussoirs, but as a simple cast dome working as a single piece." [Guastavino 1893: 72]. He then dedicates several paragraphs to explaining that a timbrel arch thrusts, but less than a voussoir arch, and continuing the reasoning applied to domes he observes that cohesive domes, which resist tension, are made of their own rings and as a consequence the dome never thrusts. However, in any constructive section of a Guastavino dome there is a metallic hoop to resist this thrust. Numerous drawings of such metallic elements exist in the Guastavino archives and the tension rings are present in every dome section of the Guastavino files in the Avery Library.

The treatise contains many other observations on the structural functioning of timbrel vaulting, and some of them demonstrate a deep understanding of the structural behaviour of timbrel vaults. But the text is also, as we have seen, filled with inconsistencies as a result of wanting to apply, in any case, his cohesive theory. After giving the reader an enlightening observation about an aspect of the structural behaviour of some element, he begins again with his contradictions and his dubious assertions.

Theory and practice in the work of Guastavino

Guastavino's cohesive theory, often incorrect and with numerous contradictions, leads one to ask: how is it possible that he was one of the greatest builders of masonry vaults and domes? The enormous variety of constructive solutions, the genius and mastery exhibited, the audacity to design unprecedented domes, are all in stark contrast to the primitive character of his theory. How is this possible?

On one hand, Guastavino began from the reference point of monolithic behaviour, cohesion, tensile and bending strength: all marks of the dominant mode of thinking in the time period in which he lived. The second half of the nineteenth century was the period of the development of elastic theory, and it

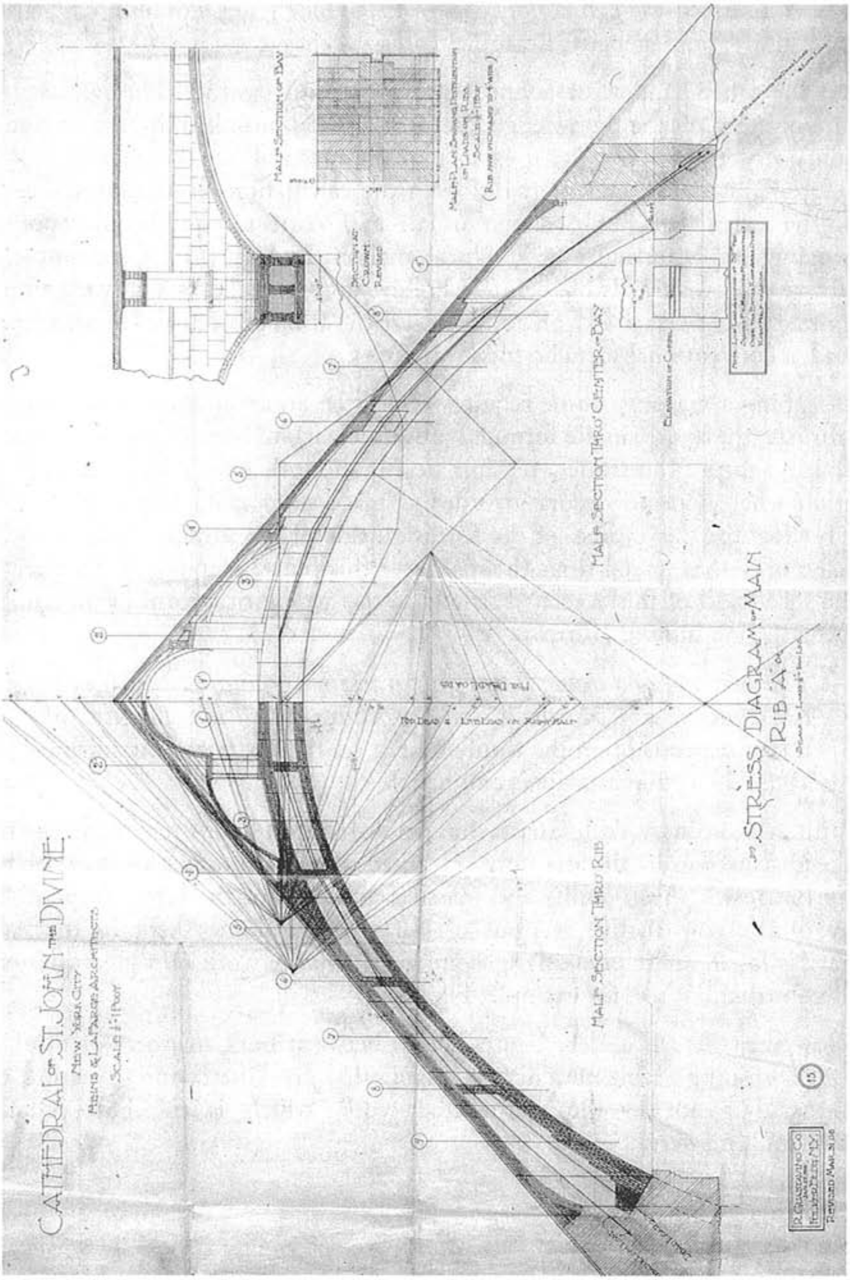

Figure 10. Graphical equilibrium analysis of the projected hollow timbrel arches for the roof of the nave of St. John the Divine (1892-1932). In the end, the arches were not constructed and the nave was covered with metallic trusses (Avery Library, University of Columbia)

easily incorporated the earlier concepts of Espie (changing monolithic behaviour for continuity, homogeneity, isotropic materials, etc.).

On the other hand, Guastavino was a great vault builder. He possessed the intuition born of the knowledge that the crucial problem in the design of masonry structures is not the resistance of the material but the geometry of the structure. This is the ancient tradition in the calculation of structures. Besides, when he calculated he used the usual, and correct, equilibrium approach employing simple formulae or graphical analysis. In Figure 10, for example, we see the graphical analysis of the great hollow timbrel arches which were to have supported the brick roof of St. John the Divine (this project was not executed; instead, a conventional metallic truss was built).

Designing a masonry dome requires very little: an approximate calculation of the thrusts (the approximate formulas are sufficient), dimensioning of the system of counter-thrust (buttresses, tension ties, or hoops), and a knowledge of the location where tension occurs in order to place supporting diaphragms or fill, which allow for the 'escape' of the forces outside of the surface of the dome. In the case of domes, metal rings (hoops) serve this purpose and vary the direction of the forces. All of this is related strictly to the geometrical form of the vaulting and Guastavino himself affirms:

> *The material of a dome is not only working by compression, but in consequence of its form, it is also working by tension, because* the thrust depends upon the form and not on the material [Guastavino 1893: 75-76; Guastavino's emphasis).

With extraordinary skill, Guastavino employed iron hoops to control the flow of the thrusts within the masonry. He used also many other devices, such as flying buttresses, dwarf vaults and massive cornices, for the same purpose. The study of the constructive sections of Guastavino's domes kept in the Avery Library is fascinating. These designs are evidently the work of a great master of vault construction; see, for example, Figure 11.

However, there is a clear contradiction between both manners of thinking, and the ensuing 'schizophrenia' is manifested in Guastavino's writing and speaking, but not in the constructed work, which is the best proof of Guastavino's mastery.

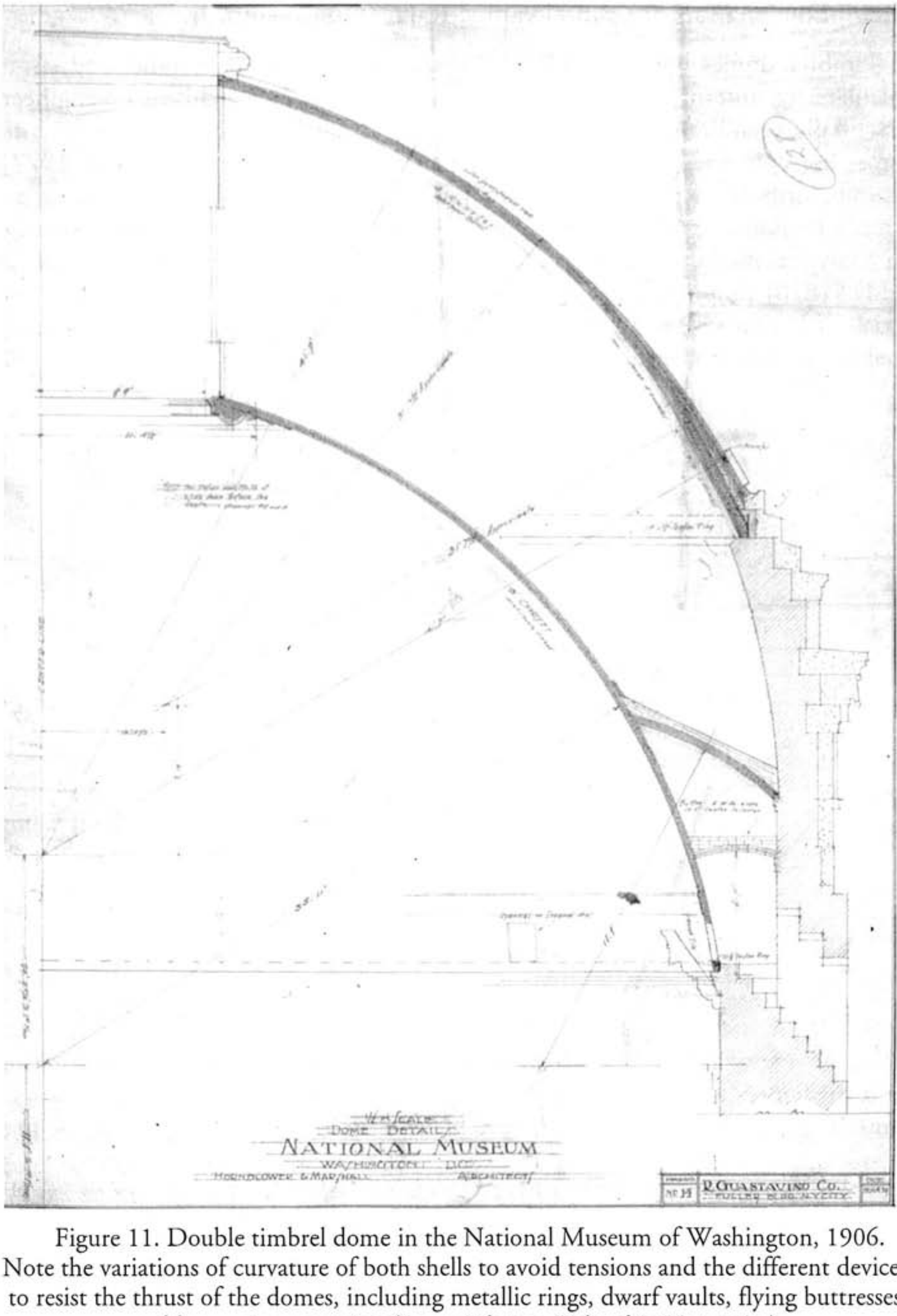

Figure 11. Double timbrel dome in the National Museum of Washington, 1906. Note the variations of curvature of both shells to avoid tensions and the different devices to resist the thrust of the domes, including metallic rings, dwarf vaults, flying buttresses and heavy stone cornices (Avery Library, Columbia University)

Membrane analysis of timbrel vaults: Rafael Guastavino, Jr.

Timbrel domes are very thin shells and the use of membrane analysis to calculate the internal forces seems obvious today for any architect or engineer. Essentially membrane analysis is an equilibrium analysis where all the internal forces are contained within the middle surface of the dome [Heyman 1977]. Simple formulae for membrane analysis of domes of revolution were given already by Rankine in 1858 [Rankine 1864: 265-8]. Schwedler [1866] developed an analytical method for trussed domes that could be extrapolated to thin shells. Eddy [1878] proposed the first graphic method, which was popularised in two articles by Dunn [1904 and 1908]. Eddy's method permits the approximate analysis of domes of revolution of any form (Figure 12).[10]

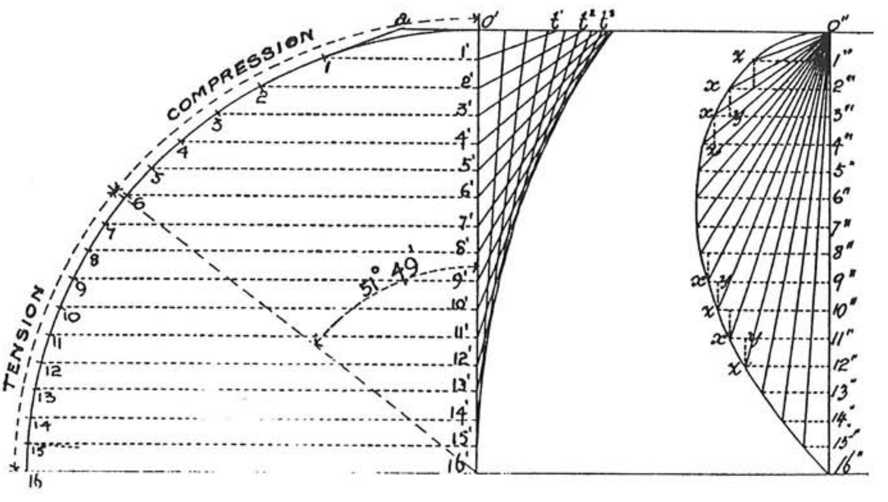

Figure 12. Eddy's graphical method for the membrane analysis of metal or masonry domes [Dunn 1904]

Rafael Guastavino, Jr. (1873-1950) worked with his father in the business from the age of fifteen. He received a "medieval" training, living and working with his father like an apprentice at the feet of a master. In addition, he taught himself art, architecture and structures in his spare time. It is most probable that

[10] The history of this method is interesting. Eddy's book was translated into German, *Neue Constructionen aus der graphischen Statik* [Leipzig, 1880]. Föppl [1881] used it without citation. Forty years later, Dischinger [1928] explained it as an analytical graphical method for the calculation of forces in thin shells of any form, again, without citing the origin. From that point on, it appeared in many manuals.

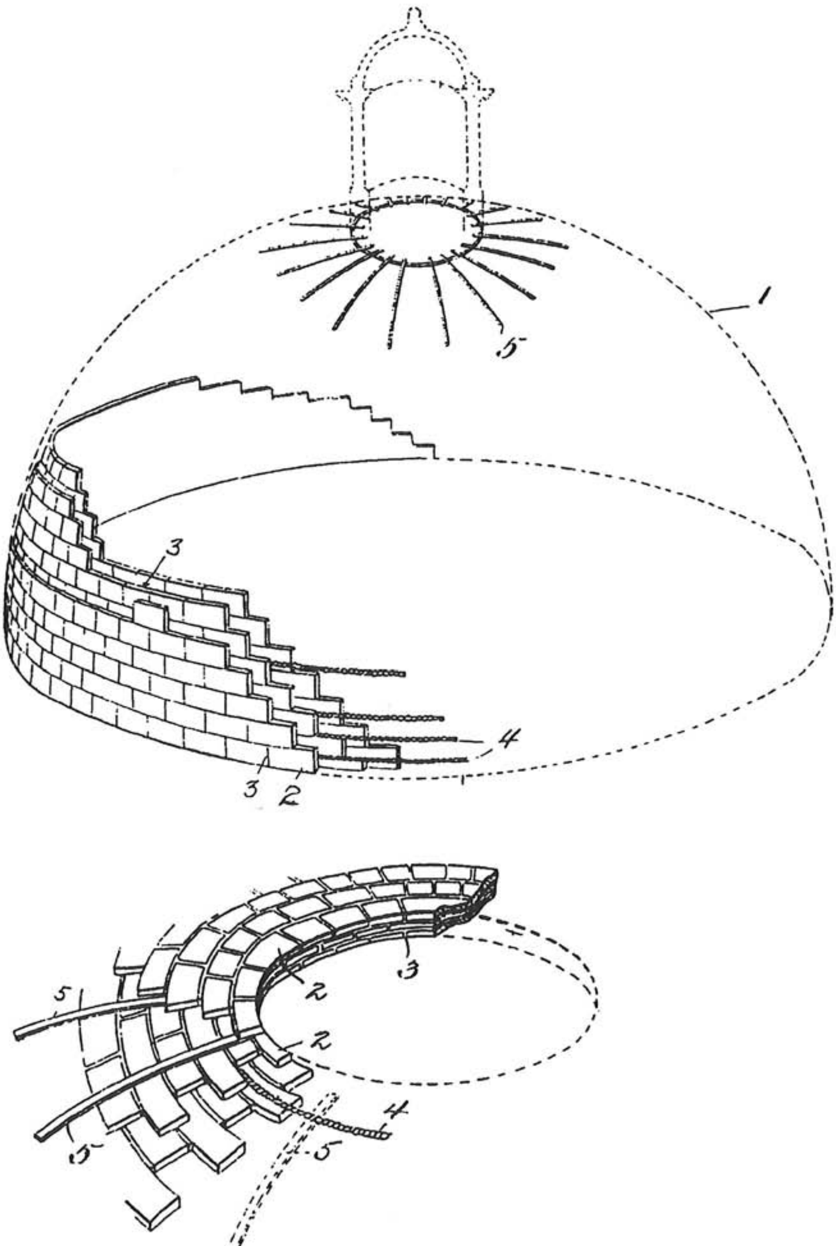

Figure 13. Placement of metal reinforcing in timbrel domes. (Guastavino, Jr., Patent, 1910)

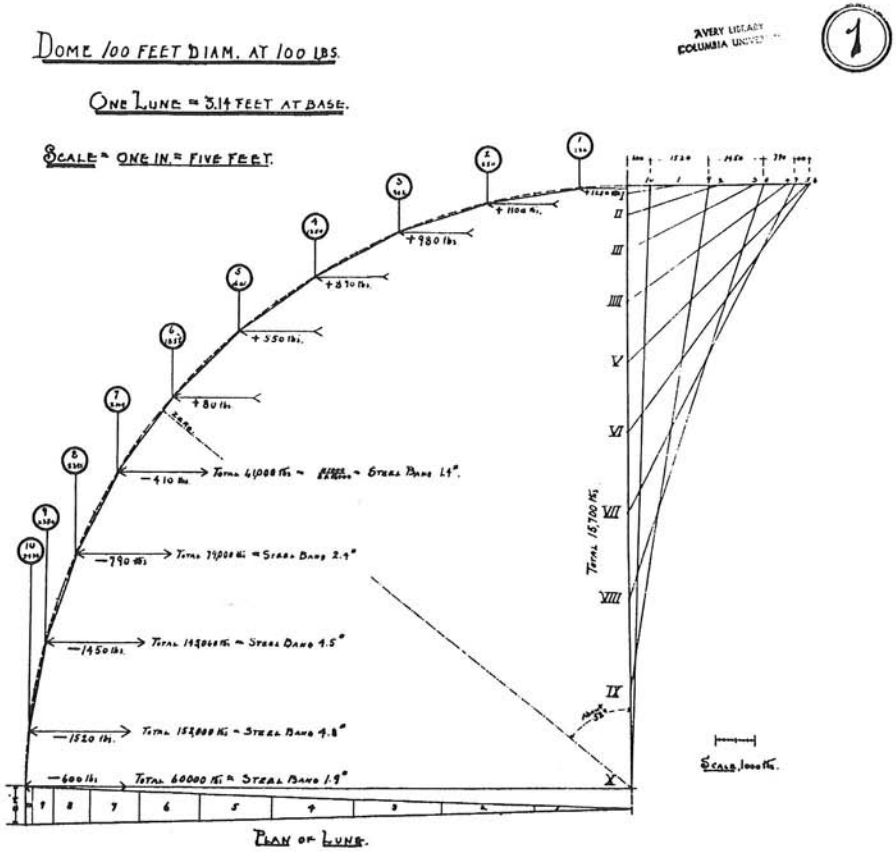

Figure 14. Graphical analysis of a thin dome of with a span of 100 ft: Plan of lune (Avery Library, Columbia University)

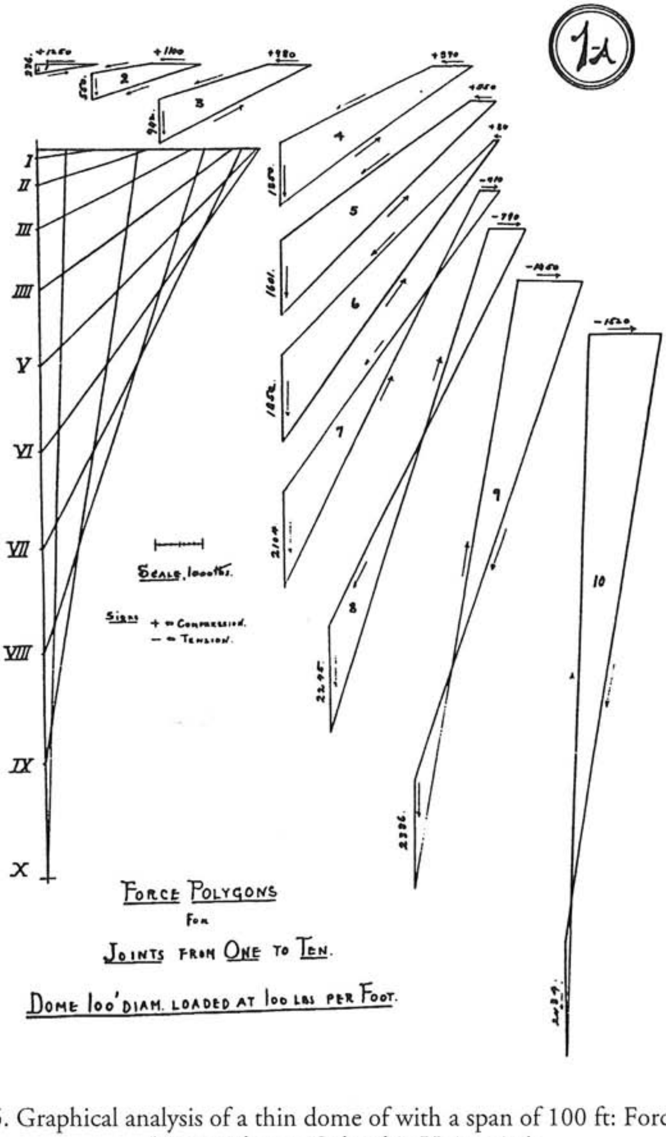

Figure 15. Graphical analysis of a thin dome of with a span of 100 ft: Force polygons (Avery Library, Columbia University)

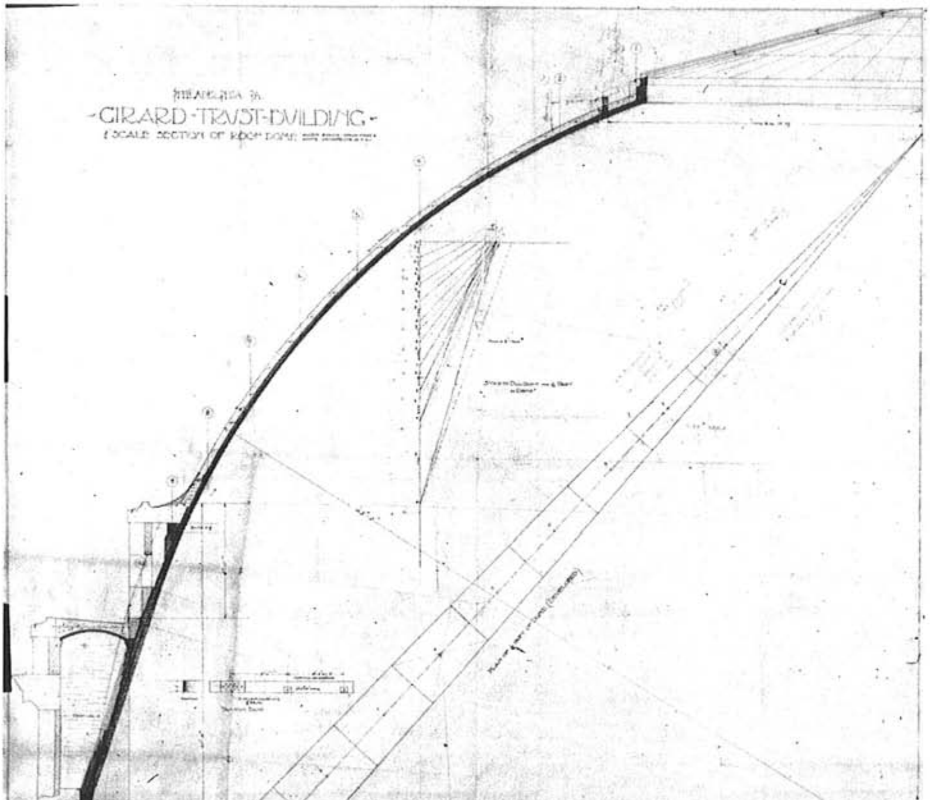

Figure 16. Design of tension-free timbrel dome. Note the change of curvature below the point of zero-stress, the horizontal component of the thrust remaining constant below; see the force polygon. Dome of the Girard Trust Building 1905-1907; 101 ft span (31 m) (Avery Library, Columbia University)

he read Dunn's contribution and decided to apply the method to the analysis of timbrel domes. In particular, he wanted to estimate the tension stresses so that it would possible to calculate and place iron reinforcement. There are two critical places: at the oculus when there is a lantern and at the base (below 52° from the top in a closed hemispherical dome). The method allows one to locate the extension of the tension zones and provide reinforcement easily. In fact, Guastavino, Jr. patented this idea in 1910 (Figure 13).

Guastavino, Jr. made this analysis for many domes, and in particular, for the great temporary dome of St. John the Divine, where metallic reinforcement was placed. In Figures 14 and 15 is shown what appears to be one of the preliminary calculations for a dome with a span of 100 ft. Guastavino, Jr. used the modified version of Dunn [1904]. It is interesting that on the second page, shown in Figure 15, the force polygon is "exploded", in order to understand better the method. It may have been an example for self-study.

Besides, Eddy's observation, republished by Dunn, that from the appearance of tension the thrust remained constant (in a dome constructed with unreinforced masonry), supplied a method for the design of domes without tension. The upper part was a spherical shell, and from the location where tension appeared, the geometry of the lower part of the dome could be traced from the force diagram to give the form of a dome without any tension. Then no reinforcement is needed and the tension hoop rings at the base may easily be calculated. The Guastavinos made extensive use of this discovery in dome design (Figure 16); see also numerous examples in Huerta [2001b: 303-313]. In fact, the approach is better than the complete catenary approach (for example of Gaudí); the dome has a simple geometrical form in the upper part and only deviates from it when it is needed in the lower parts.

Guastavino, Jr. did not publish any papers, but he gave several seminars.[11] In fact, he was profoundly affected by the decline of masonry construction, which he had learned and practised his entire life. To maintain the business, he researched various chromatic possibilities for brick, and above all, he made pioneering research into acoustic materials by collaborating with the leading expert of the period, W.C. Sabine. In the 1930s, he competed with the rise of thin shell construction in concrete, and his interest in this subject is well documented at the Avery Library of Columbia University. Finally, he built a timbrel dome for the Buhl Planetarium of 1938, though such domes were mostly built in reinforced concrete following Dischinger's pioneering work. But this was no longer the age of masonry construction, and the company survived into the

[11] In the Guastavino archive of the Avery library, a manuscript is preserved for a magazine article of 1929 and the text of a conference seminar given around 1914.

1960s by developing acoustic materials and building vaults for the last historicist buildings.

Elastic analysis: Domenech, Bayó, Terradas

The table produced by Lanza for Guastavino's *Essay* of 1893 is probably the first evidence of elastic analysis of a timbrel arch. In fact, by the end of the nineteenth century elastic analysis was considered the best approach for masonry arches. The discontinuity and heterogeneity of the masonry, the difficulty in obtaining the elastic constants, the movements during construction, the cracking, etc., were evident, and some engineers were conscious of the dubious character of elastic assumptions applied to masonry arches (see for example [Swain 1927: 425]), but the force of elastic ideas was so great as to overcome any resistance.

Elastic ideas of continuity, tension and bending strength, fit well with Espie's monolithism and Guastavino's cohesion. The only, fundamental, difference is that elastic arches do thrust. The emphasis, then, is in the bending and tension strength of the timbrel vaults. Following Bergós, Gaudí made some calculations to take into account the bending strength of timbrel arches. However, it should be kept in mind that Gaudí published nothing on these matters and that the indirect testimony of Bergós could be biased by his own ideas. A question arises: If Gaudí believed in the bending strength of masonry, why did he use bending-free catenary models? (For the hanging models of Gaudí, see Tomlow [1989].)

It was José Domenech y Estapá who first considered the necessity of taking into account the resistance to bending moments. For Domenech there was no doubt that the only explanation for the success of the thin timbrel vaults came from its capacity to resist bending moments that could cancel the horizontal thrust:

> *The mechanical secret to the construction of these vaults...is not in limiting the calculation of the compressive strength of the materials used, but in taking advantage of the tensile resistance and transverse strength offered by our bricks combined with lime or cement mortar.*
>
> *Utilizing these two strengths, the Catalonian builder could dare to subject his vaults to loads which are unthinkable in others [structures]...always with a small horizontal thrust at the supports, and in some cases reducing the thrust to zero* [Domenech 1900: 38-39].

Once again, the rigid monolithic idea of Espie appears, together with the myth of the absence of thrusts and the cohesive resistance to bending. Later, however, Domenech makes a lucid analysis of the function of timbrel arches, taking as an example the case of a uniform load, in which the line of thrusts is a parabola. He observes that if the directrix of the arch coincides with the line of thrusts (i.e., the arches are exactly parabolic), then there would only be compression, but he leaves this aside. Domenech continues by explaining the method of finding the bending moments and shear forces for a given line of thrust. Finally, he discusses the problem of the position of the line of thrusts, considering the possibility of cracking ("joints of rotation") in an arch (Figure 17).

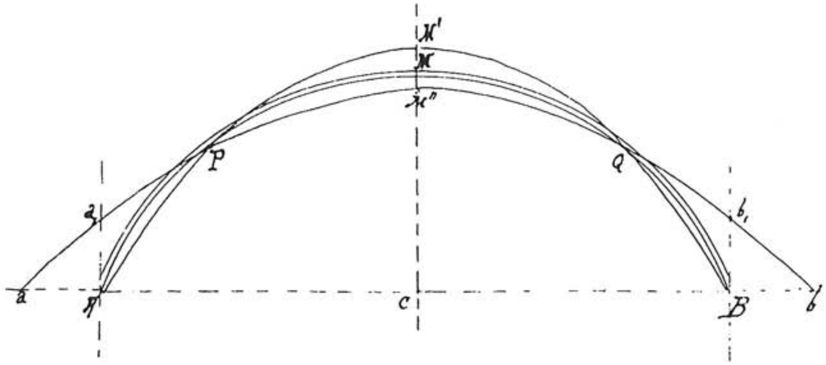

Figure 17. Possible positions of the thrust line in a timbrel arch [Domenech 1900]

Domenech's drawing shows the line of thrust outside the masonry and producing bending moments. In fact, Domenech commits a frequent error: identifying the structure only as the vault, forgetting the fill over the supports and the transverse diaphragms which support the vault. These elements *are also structural* and offer alternative paths for the thrusts to reach the supports. (The supports may be buttresses or masonry walls, a horizontal metal beam supporting ties, etc.) In any case, the situation cannot possibly be maintained over time, due to the low tensile resistance of the masonry, its fragile character, which allows cracking easily and, above all, the inescapable necessity of cracking to adapt to small displacements of the abutments.

Martorell wrote about similar considerations on the bending resistance and the resulting reduction of thrusts:

> *The methods of graphical mechanics used generally, applied to brick arches and in a special way to timbrel arches, give results which are less favourable than the corresponding reality . . . The cohesion, the rigidity of timbrel vaults, greatly lowers the thrust and at the same time, allows them to be built in implausible forms, as if they were metallic shells* [Martorell 1910: 143].

Martorell alludes to the various positions of the line of thrust, and implicitly, to the appearance of bending moments, highlighting the necessity for tests allowing the calculation of the "coefficients used in calculations to evaluate the bending resistance and the transverse forces in the timbrel vaults".

Jaime Bayó [1910] is the first in Spain to propose a proper elastic analysis for timbrel vaults. In his article, he equates them with metal arches (two-hinged), criticising the traditional methods for the calculation of voussoir arches:

> *To calculate this [timbrel] vault... by determining the line of thrust of a voussoir arch, is born of an error, which supposes that it only works in compression. This is not the case, as it works also in tension, being like any metal surface it can support bending* [Bayó 1910: 165].

For Bayó the timbrel vaults thrust, but this thrust corresponds to that of a two-hinged, elastic arch. He tries to find what he calls "the funicular of the elastic forces", that is, the line of thrust that is in equilibrium with the loads, and also complies with the compatibility conditions of elastic deformations. Bayó gives the formulas with the usual integrals, and later, he explains a graphic solution method, applying it first to symmetrical arches of constant or variable thickness, and then to asymmetrical arches. He also explains how to calculate the tensile and compressive stresses, and cites the experimental tests by Guastavino as a current reference to consider the values for allowable stress: 1.5 N/mm^2 in compression, 0.4-0.5 N/mm^2 in tension, and 0.6 N/mm^2 in shear. The article finishes with some considerations on the design of timbrel vaults, in which he recommends adjusting the thickness (number of brick layers) according to the bending stresses. He observes that in the case of the flat vaults, the thrust can be calculated as if it were made of voussoirs, working only in compression, but if they are higher it is precise to adapt the form to the line of thrust. He finishes the article proposing a method to design timbrel vaults of any shape:

> *If it is desired to construct equilibrated vaults or of equal resistance, that respond to the design suggested by the imagination of an artist, one should proceed as shown in the figure... ...after determining the funicular of the elastic forces, the thickness of the vault is given in relation to the value of the bending moments* [Bayó 1910: 184] (Figure 18).

We have no evidence that Bayó or any other architect ever constructed a timbrel vault (or any other masonry structure), with this form and thickness, but the drawing clearly shows the strong belief in tensile resistance, in elastic calculations, and in the cohesive properties of timbrel vaults.

Finally, it must be noted that the calculation of timbrel arches corresponds to the most elementary example of timbrel vault construction. Bayó does not mention the calculation of the more complex and most common forms, such as the crossing vault, barrel vault, ribbed vault and domes. For the calculation of internal forces in these cases, the only viable approximation was the equilibrium calculations using graphic statics or hanging models and this was the common practice.

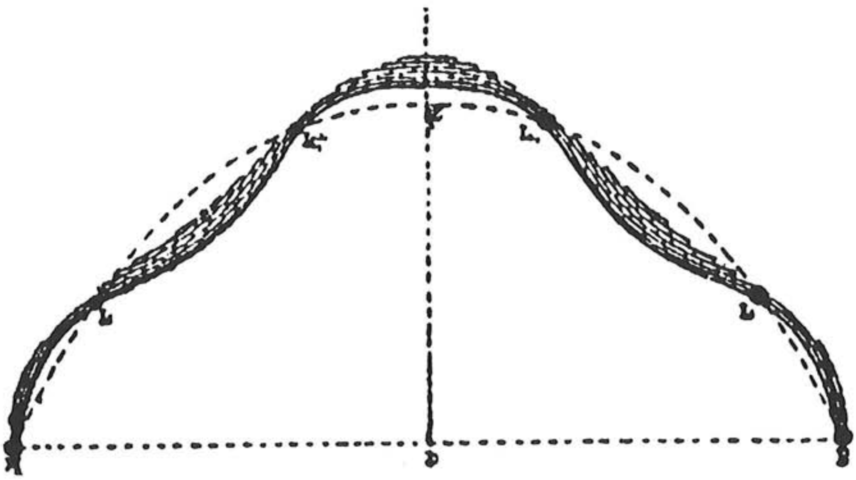

Figure 18. Timbrel vault of a peculiar form (impossible to construct in reality) where the thickness is prescribed according to the magnitude of the bending moments resulting from an elastic analysis [Bayó 1910]

Consolidation of the elastic-cohesive focus: vaults 'impossible to calculate'

The cohesive ideas formulated first by Espie, taken up, expanded and spread by Guastavino, and later adopted by elastic theory, were converted into a dogma. In a book of 1910 on the philosophy of structures, Cardellach frames the timbrel vaults under cohesive construction, highlights the capacity to resist bending and, like Bayó, insists on the infinite variety of forms that can be constructed.

Esteve Terradas, the great Spanish engineer and mathematician, was the first to try an elastic analysis of a more complex timbrel vault: the vault of a staircase. The contribution of Terradas has been analysed in detail by Rosell and Serrá [1987]. In this context it must be stressed that Terradas's study had its origin in the challenge of Puig y Cadafalch, in 1919, to solve the problem of the calculation for timbrel vaults. As Rosell and Serrá write, "the habitual vaults, constructed by bricklayers 'from feeling', were considered to be impossible to calculate" [Rosell and Serrà 1987: 24]. Terradas collected his sketches, notes, and calculations in a small book called *Libreta de la volta*.[12]

Terradas tried to make an elastic analysis of the vaults and he examined well-known elastic problems, in particular that of buckling. He failed in his attempt. The planning of the elastic equilibrium equations for a spatial structure such as a vaulted staircase is very complex. The failure of Terradas served to reinforce the idea of the impossibility of calculating the forces in timbrel vaults.

Later, Josep Goday, in his 1934 discourse before the Académia Catalana de Belles Arts de Sant Jordi, made a historical survey of the calculation of timbrel vaulting. He accepts the cohesive ideas of Guastavino and agrees with Bayó and Terradas that the only correct focus is to consider the vaults as thin, continuous, elastic shells, within the context of elastic theory. At the end of his article, Goday briefly discusses membrane analysis, but does not seem to appreciate that it is an equilibrium method which does not consider the material properties. (The membrane theory, dating from the second half of the nineteenth century, was popularised in Europe in the 1930s, principally through the theoretical and practical work of the German engineer Franz Dischinger, using it to design thin shells of reinforced concrete.)

These confirmed the idea that timbrel vaults could only be calculated as elastic, and if this task presented insurmountable difficulties, then the vaults were impossible to calculate. Eduardo Torroja, the great Spanish engineer and builder of thin concrete shells, repeated this opinion, writing, "so marvellous in its constructions, that modern theories have difficulty explaining and measuring its phenomenal resistance, so brilliantly intuited by builders of the past" [Torroja 1956]. Bassegoda, in his numerous contributions on timbrel vaulting [Huerta et al. 2001], expressed similar opinions, and more recently, Professor J. L. González [1999] considered it necessary to make a load test on a timbrel vault stair to reliably estimate its strength.

Calculations in Practice

As Rankine noted accurately in the Introductory Essay to his book on *Applied Mechanics*, if the question of theoretical science is "what are we to think?", the question in practical science is "what are we to do?" [Rankine 1858: 10]. An insufficient theory, real or imagined, has never deterred the builders who have

[12] Professor Rosell provided me with a photocopy of this book and volunteered his time to speak with me on various aspects of Terrada's work. For all of this, I am extremely grateful to him.

applied the available tools of the time period. Thus, while the engineer-scientists argued over the impossibility of calculating the forces in timbrel vaulting, the builders continued constructing and the architects or engineers made simple calculations to determine the dimensions of the principal elements: the thickness of the vaults, and the sizing of the systems to resist the thrust.

The builders' clear belief in the thrust of timbrel vaults can be demonstrated by their use of systems to counter thrust, which are always present. As we have seen, traditional rules for buttress design for timbrel vaults have existed. The French engineers of the nineteenth century made equilibrium calculations, as did the Guastavinos, though the hypotheses behind the formulas were in direct opposition to the cohesive theory. Luis Moya [1957], the last great builder of timbrel vaults, acknowledged the insufficiency of calculations owing to the lack of data on the elastic constants of timbrel vaults, but later he made, or directed another to make, equilibrium calculations based on the line of thrust to design and build his astonishing vaults.

Bosch [1947] spoke in favour of membrane analysis but for practical cases proposed an ingenious system (inspired, no doubt, by nineteenth century manuals on the theory of vaulting) to calculate the thrust of timbrel vaulting, by cutting the vault into a series of arches. He imagined the existence of virtual crossing ribs which support a series of parallel arches between the ribs. Again, this is an equilibrium method that seeks to find one possible state of compression within the masonry.

Bergós [1936; 1953; 1965] dedicated several decades to studying the mechanical properties of masonry walls and timbrel vaults. He tested timbrel arches of various sizes (up to 3.2 m in span), trying to justify the application of elastic theory. But in the examples of practical calculation that appear in his books he uses graphic methods of thrust lines, that is, equilibrium methods.

Angel Pereda Bacigalupi [1951] published one of the last books on the calculation of timbrel vaulting. Like Bayó, he supposed two-hinged arches on rigid supports, and calculated them with the usual equations for elastic arches. Vaults were often built with tension ties to take the thrust, but the deformation of this tie was not considered in the calculation, even though it would lead to significant bending moments. In fact, Pereda realized that an elastic calculation could not pretend to account for the flexural resistance of timbrel vaulting. He explicitly looks for the thickness so that the line of elastic thrusts is contained within the middle third of the section. To accomplish this, Pereda lowered the admissible tensile stress, demonstrating a better knowledge of the material properties than his earlier predecessors working with elastic calculations.

The use of Finite Element Methods

Today, the finite element method (FEM) has been applied to the analysis of timbrel vault structures. Gulli [1993;1994; 1995] made tests on barrel vaults and later, implemented finite element methods to carry out elastic calculations. The finite element method, like traditional elastic calculations, considers the masonry as a continuum with certain elastic properties, which require assumptions about the support conditions. These assumptions about the supports and the material, together with static equilibrium, form a system of equations that give a unique solution. This focus presents various problems. In the first place, the system of equations is extremely sensitive to small variations in the support conditions. For example, a small settlement or rotation of one of the supports, imperceptible to the eye, will give a large variation in the system of internal forces (and the analyst can use a FEM program to verify this point). In the second place, timbrel construction is far from a continuum and is frequently cracked. The use of FEM programs allow a non-linear analysis, which improves the model, but is still highly sensitive to changes in the support conditions, the load history of the structure, the formation of cracks in unexpected locations, etc. In summary, the results from an elastic analysis or the FEM have little significance, and are of no assistance in understanding the structural behaviour of the timbrel vault or masonry structure in question.

Conclusion : The timbrel vault as a masonry vault

Timbrel vaults are masonry vaults, with a good strength in compression, a low tensile strength and the possibility of cracking, forming 'hinges', due to the impossibility of sliding. In fact, in a hyperstatic masonry structure the formation of cracks is inevitable, and traditional timbrel vaulting shows the same pathologies of stone or brick vaulting.

It is true that the tensile resistance also allows a certain amount of bending resistance. For example, a mason can walk on a thin timbrel arch. To explain the resistance to higher loads or over longer periods of time, one must look to the other resisting elements. Thin walls, or lateral diaphragms, as well as a solid fill, which forms the base for a floor, are actually part of the resisting structure and help to resist moving loads. The principle is always the same: to give a sound escape route for the forces when necessary, or to load the structures so that the line of forces is always contained within the masonry.

The possibility of perforating a vault without collapse, which has been cited since Espie as a characteristic of the cohesive structure, is also true of other masonry vaults. Every so often a pinnacle falls from a buttress and perforates a Gothic crossing vault without causing collapse.

The 'cohesive' character is not relevant from the structural point of view, but is important from the constructive point of view. It allows for the construction without centering, using only light auxiliary elements to control the form. In addition, timbrel vaults present some resistance to bending, which permits the passage of light loads during construction, making the process even easier.

Summing up, the fundamental statements about the 'masonry' material (high compressive strength, low tensile strength and no sliding) apply also to timbrel vaults. Professor Heyman has systematised these properties to include masonry structures within the more general framework of Limit Analysis. From his first article in 1966 until today he has clearly illustrated the theory by applying it to basic structural elements: buttresses, domes, crossing vaults, spires, towers, bridges, etc. (The articles have been compiled in Heyman [1995], as well as an overview of his work in [1997].)

Within the frame of Limit Analysis, the Safe Theorem validates the approach of equilibrium: if we can find an equilibrium solution for the masonry structure with the material working in compression, then the structure is safe. The power of the theorem lies in that we may 'choose' the equilibrium solution. If the analyst can find a situation of equilibrium in compression. the structure will be able to as well. In fact, the analyst will study only some of the infinitely many equilibrium states possible in a hyperstatic structure.

The equilibrium analysis of the old theory of vaults is therefore perfectly correct, and lies within the scope of Limit Analysis [Huerta 2001].[13] The simplified formulas of Guastavino, the graphic analysis, Gaudí's use of hanging models, and membrane analysis of compression states by Guastavino, Jr., are all correct. Elastic analysis 'in compression', like that of Pereda cited above, are also correct. To state it more clearly, they are 'safe': a structure designed on the basis of them will not fall and the same methods can be used to measure their safety. In fact, there could be no other conclusion for structures which have survived for more than a century, and this experimental demonstration is conclusive.

Even more, the traditional proportional rules for the design of vaults and buttresses (like those of Fray Lorenzo) are also essentially correct [Huerta 1990; 1999]. The problem of the safety of a masonry vault, whether of stone, brick, mass concrete or a timbrel vault, is a problem in the geometrical form of the structure. The stable forms contain lines of thrust in equilibrium with the applied loads. The traditional rules codify these forms and their use is rational

[13] The approach of equilibrium has been recently applied to the calculation of the thrust of timbrel groined vaults by Fortea and López [1998].

and correct. (Of course these rules are particular for each type of structure: a Gothic buttress would not support the thrust of a Roman vault.)

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